

Observational Consequences of Quantum Cosmology

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ABSTRACT

Our universe is born of a tunnelling from nothing in quantum cosmology. Nothing here can be interpreted as a state with zero entropy with the viewpoint of a dual CFT proposed by Verlinde. As a reliable modification of the Hartle-Hawking wave function of the universe, the improved Hartle-Hawking wave function proposed by Firouzjahi, Sarangi and Tye gives many interesting observational consequences which we explore in this paper. Fruitful observations are obtained for chaotic inflation, including a detectable spatial curvature and a negligible tunnelling probability for eternal chaotic inflation. And we find that the tensor-scalar ratio and the spatial curvature for brane inflation type models should be neglected.

In the last few years, the cosmological observations [1,2] are successful in making the history of our universe clearer. Recent expansion of our universe is accelerating, due to the dark energy with negative pressure. As a simplest candidate for the dark energy, the cosmological constant can fit today's data very well. Hot Big Bang must happen in the early universe. In order to solve the flatness, horizon and monopole puzzle etc. in Hot Big Bang model, a procession with the expansion of the universe accelerating, named inflation, should happen before the Hot Big Bang. Inflationary models also naturally offer the seeds for the formation of the large scale structure of our universe and the temperature fluctuations in CMB.

However the initial condition for our universe is still unknown. Why and how inflation happens before Hot Big Bang are still mysterious in modern physics. Until now, the most attractive idea about the origin of our universe is the Hartle-Hawking no-boundary wave function Ψ_{HH} of the universe [3] which says that our universe is born of a tunnelling from nothing (here nothing means a state without any classical spacetime [4]). A number of other conjectures on the origin of our universe are also discussed in [5]. In general, we believe the quantum theory of gravity can give us more insights on the origin of the universe. Unfortunately, we still don't know the full quantum theory of gravity. String theory is the only self-consistent candidate for quantum gravity. Recently some advancements took place in string theory, notably string compactifications [6], which provide many stable or meta-stable vacua with lifetimes comparable to or larger than the age of our universe. The number of such vacua may be $\mathcal{O}(10^{100})$ or larger. On the other hand, in the inflationary scenario, our observed universe is a tiny patch of the whole spacetime in the early universe and the whole space-time may contain many disconnected regions which will grow up to many subuniverses with different physical parameters. How to select a particular vacuum where we live is still a big open question. A line to go beyond the Anthropic principle is to reliably calculate the tunnelling probability from nothing and pick out

the vacuum with the largest tunnelling probability (see, for example, [7]).

Hartle-Hawking wave function of the universe favors a universe with cosmological constant $\Lambda = 0$. Inflation can not happen in such a universe. It does not agree with the history of our universe. In this short note, we prefer an improved Hartle-Hawking wave function proposed by Firouzjahi, Sarangi and Tye in [8, 9]. This wave function offers a natural realization of the spontaneous creation of inflationary universe from nothing. According to the Cardy-Verlinde formula [10], the evolution of a closed universe can be described by a dual CFT and we can use it to make the meaning of nothing clearer, i.e. nothing means that the initial state of our universe corresponds to a state of CFT with zero entropy. We also use this improved wave function of the universe to investigate several typical inflationary models, such as chaotic inflation and brane inflation type model. Many interesting observational consequences are obtained.

Let us start with the action for Einstein theory in four dimensional space-time with a positive cosmological constant Λ ,

$$S_E = \int d^4x \sqrt{-g} \left(-\frac{R}{16\pi} + \Lambda \right). \quad (1)$$

We work on the unit with Newton coupling constant $G_N = 1$ in the whole note. The Euclidean metric is given by

$$ds^2 = \sigma^2 \left[d\tau^2 + a^2(\tau) d\Omega_3^2 \right], \quad (2)$$

where $\sigma^2 = 2/3\pi$ and $d\Omega_3^2$ is the metric on a unit S^3 . Since the volume of the spatial flat or open universe is infinite,¹ the tunneling probability from nothing to any one of them is suppressed and it is only possible that a closed de Sitter universe emerges. The equation of motion for the universe is given by

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{1}{a^2} = \frac{16\Lambda}{9}. \quad (3)$$

¹If the topology or the configuration of the universe is nontrivial, a flat or open universe is also possibly tunnelling from nothing, see for instance [11, 12].

The tunneling point corresponds to $\dot{a}/a = H = 0$ and our universe started with a finite size, $a = 3/(4\sqrt{\Lambda})$, not $a = 0$, which says that Hartle-Hawking wave function can help us to avoid the cosmic singularity.

Recalling the proposal by Verlinde in [10], the entropy of a $(1 + 1)$ -dimensional CFT is given by Cardy formula

$$S = 2\pi\sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)}, \quad (4)$$

which is universally valid in arbitrary dimensional spacetime if we define the central charge c in terms of the Casimir energy. Via some identifications, for instance, the entropy of this CFT with the $(n - 1)\frac{HV}{4G}$ (see [10] in details), where H is the Hubble constant and V is the volume of $(n + 1)$ -dimensional closed Friedman-Robertson-Walker universe, the Cardy formula (4) exactly turns into the $(n + 1)$ -dimensional Friedman equation for a closed universe. Or the entropy of a closed universe with the viewpoint of a dual CFT can be given by

$$S_H = (n - 1)\frac{HV}{4G_N}. \quad (5)$$

When the universe was born, the expansion rate of the universe $H = \dot{a}/a$ equals zero. Using eq. (5), the entropy of the universe equals zero as well. Thus the meaning of nothing in quantum cosmology can be interpreted as a state with entropy being zero.

According to Hartle-Hawking wave function, the tunnelling probability from nothing to a closed de Sitter universe with cosmological constant Λ is

$$P_{HH} = |\Psi_{HH}|^2 \simeq \exp\left(\frac{3}{8\Lambda}\right). \quad (6)$$

This wave function is not normalizable and picks out $\Lambda = 0$ with the largest tunnelling probability. It contradicts with the history of our universe. The size of the universe is roughly $1/\sqrt{\Lambda}$, a macroscopic size. Tunnelling is a pure quantum phenomenon and the tunnelling probability is suppressed for a quantum system with macroscopic size. Naturally we expect that the

probability for tunnelling to a universe with $\Lambda = 0$ is suppressed by the effects of decoherence and a universe with microscopic size naturally emerges. This is just the idea proposed by Firouzjahi, Sarangi and Tye to improve Hartle-Hawking wave function [8, 9]. The decoherence comes from the fluctuations of the metric. After integrating out these fluctuations, a new term which behaves just like ordinary radiation appears in the effective Lorentzian action

$$S \simeq \frac{1}{2} \int d\tau \left(-a\dot{a}^2 + a - \alpha a^3 - \frac{\nu}{\alpha^2 a} \right), \quad (7)$$

where $\alpha = 16\Lambda/9$ and ν is a constant which measures the number of perturbative modes and depends on the UV cutoff for the gravitational interaction in quantum theory of gravity. Roughly the value of ν relates to this UV cutoff M by (see [9])

$$\nu = M^4/(72\pi^6). \quad (8)$$

In string theory, this UV cutoff is naturally taken as string scale $M_s = 1/\sqrt{\alpha'}$, a high energy scale. Since we do not know string scale exactly, we can not determine the value of ν . In particular, in the framework of KKLT [6], the effective string scale in different throats can take different values, which depends on the value of warp factor. It is natural that the UV cutoff in eq. (8) is lower than the Planck energy scale M_{pl} and then $\nu \leq 10^{-5}$. Here we will fix it by using the observational data.

Now the equations of motion following from the improved action (7) are given by

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{a^2} = \alpha + \frac{\nu}{\alpha^2 a^4}, \quad (9)$$

$$\frac{\ddot{a}}{a} = \alpha - \frac{\nu}{\alpha^2 a^4}. \quad (10)$$

The initial size of the universe is

$$a = \frac{1}{\sqrt{2\alpha}} \left(1 + \sqrt{1 - \frac{4\nu}{\alpha}} \right)^{1/2}. \quad (11)$$

Here we require $\alpha \geq 4\nu$, which is just the condition for the existence of the instanton solution. The reason we choose this solution (11) is that the right hand side of eq. (10) is positive and the expansion rate of the universe can speed up from zero after the universe was born. The author of [8, 9] also calculate the tunnelling probability according to this improved action as

$$P \simeq \exp \mathcal{F} = \exp \left(\frac{3}{8\Lambda} - \frac{27\nu}{32\Lambda^2} \right). \quad (12)$$

We need to remind that the instanton solution is destroyed and the tunnelling probability equals zero when $\Lambda < 9\nu/4$. Since the UV cutoff is a high energy scale, the effective cosmological Λ is large and inflation can naturally occur.

In the following, we will investigate the consequences for several typical inflation models by using this improved tunnelling probability (12). Here we make the loose-shoe approximation [13], $\phi = \text{const}$ and the potential $V(\phi)$ term is equivalent to a cosmological constant. Therefore eq. (12) is recovered.

First we work on the inflation model with only one fine tuning parameter, for example, chaotic inflation model [14]. The potential for the inflaton field ϕ is given by

$$V(\phi) = \lambda\phi^n, \quad (13)$$

where $n \geq 1$. The parameter needed to be fine tuned is the coupling constant λ which can be fixed by the amplitude of the power spectrum. Or equivalently, there is no free parameter at all if we take the amplitude of the power spectrum as an input. The equations of motion in the slow rolling limit can be written as

$$H^2 \simeq \frac{8\pi V(\phi)}{3} = \frac{8\pi}{3}\lambda\phi^n, \quad (14)$$

$$3H\dot{\phi} \simeq -V'(\phi) = -n\lambda\phi^{n-1}. \quad (15)$$

The number of e-folds before the end of inflation is

$$N = \int H dt = \int \frac{H}{\dot{\phi}} d\phi \simeq \frac{4\pi}{n}\phi_N^2. \quad (16)$$

Or equivalently, the value of ϕ , namely ϕ_N at the number of e-folds N before the end of the inflation is

$$\phi_N = \sqrt{\frac{n}{4\pi}} N. \quad (17)$$

When $\phi = \phi_N$, the potential becomes

$$V(\phi_N) = \tilde{\lambda} N^{\frac{n}{2}}, \quad (18)$$

with

$$\tilde{\lambda} = \lambda \left(\frac{n}{4\pi} \right)^{\frac{n}{2}}. \quad (19)$$

The tunnelling probability (12) can be obtained as

$$P \simeq \exp \mathcal{F} = \exp \left(\frac{3}{8\tilde{\lambda} N^{n/2}} - \frac{27\nu}{32\tilde{\lambda}^2 N^n} \right). \quad (20)$$

Maximizing \mathcal{F} in eq. (20), we obtain

$$N = N_m = \left(\frac{9\nu}{2\tilde{\lambda}} \right)^{2/n}. \quad (21)$$

Now the tunnelling probability is given by

$$P \simeq \exp \mathcal{F} = \exp \left(\frac{1}{24\nu} \right) = \exp \left(\frac{3}{16\tilde{\lambda} N_m^{n/2}} \right). \quad (22)$$

The amplitude of the power spectrum δ_H is

$$\delta_H = \frac{1}{5\pi} \left| \frac{H^2}{\dot{\phi}} \right| \simeq \left(\frac{2^7}{n \cdot 3 \cdot 5^2} \right)^{\frac{1}{2}} \tilde{\lambda}^{\frac{1}{2}} N^{\frac{n+2}{4}}, \quad (23)$$

where N evaluates the number of e-folds when the fluctuations just stretched outside the Hubble horizon during the period of inflation. The COBE normalization is $\delta_H \simeq 2 \times 10^{-5}$ for $N \sim 60$. If $n = 4$, we find $\tilde{\lambda} \simeq 4.3 \times 10^{-15}$ and $\mathcal{F}_{N_m} \simeq 4.4 \times 10^{13}/N_m^2$. For a roughly spatial flat universe, the total number of e-folds should not be smaller than 60. Using eq. (21), we can get a low bound on the parameter ν as $\nu \geq 3.4 \times 10^{-12}$ and an upper bound on the tunnelling probability with $\mathcal{F} \leq \mathcal{F}_{60} = 1.2 \times 10^{10}$. Combining with eq. (8),

we can get a low bound on the UV cutoff in eq. (8) as $M \geq 0.02M_{pl}$, where $M_{pl} = G_N^{-1/2} = 1.2 \times 10^{19}\text{Gev}$ is the Planck energy scale in four dimensions. In string theory, we take this UV cutoff as string scale, which means that the string energy scale should not be lower than $0.02M_{pl}$. Here we need to keep in mind that a successful realization of a chaotic inflation in string theory is still not known.

For a given ν , the distribution for the number of e-folds can be written as

$$P \simeq \exp\left(\frac{3}{8\tilde{\lambda}}\left(\frac{1}{N^{n/2}} - \frac{N_m^{n/2}}{2N^n}\right)\right) \sim \exp\left(-\frac{(N - N_m)^2}{2\sigma^2}\right), \quad (24)$$

which is a Gaussian distribution with variance

$$\sigma = \sqrt{\frac{32\tilde{\lambda}}{3n^2}} N_m^{1+\frac{n}{4}}. \quad (25)$$

Here we drop out a whole constant factor in the last step and take a limit with $|N - N_m| \ll N_m$ in eq. (24). For $n = 4$ and $N_m = 60$, the variance roughly equals 2×10^{-4} . Therefore, for a given ν , the number of e-folds must be N_m given by eq. (21) at a high level of statistical significance.

As a generic phenomenon, eternal inflation is common to a very wide class of inflation models [15]. During the period of inflation, the evolution of the inflaton field ϕ is influenced by quantum fluctuations, which can be pictured as a random walk of the field with a step $\delta\phi \sim H/2\pi$ on a horizon scale (Hubble scale H^{-1}) per Hubble time $\Delta t \sim H^{-1}$. During the same epoch, the variation of the classical homogeneous inflaton field rolling down its potential is $\Delta\phi \sim |\dot{\phi}|H^{-1}$. If the classical variation is smaller than the quantum fluctuations, the role played by fluctuations becomes significant and the inflaton field can walk up the potential, rather than roll down the potential in some spacetime regions. If the eternal inflation happened, the wave function of the universe can not help us to select a particular vacuum where we live. The eternal inflation provides a natural arena for the Anthropic principle. Different parts of spacetime can be characterized by different effective values of nature. The values we observe are determined by Anthropic selection [16].

We can estimate that the eternal chaotic inflation with potential (13) happened when $\delta\phi \geq \Delta\phi$, or

$$\phi \geq \phi_* = \left(\frac{3n^2}{128\pi\lambda} \right)^{\frac{1}{n+2}}. \quad (26)$$

If inflation started with $\phi = \phi_*$, the total number of e-folds will be

$$N_* = \frac{4\pi}{n} \left(\frac{3n^2}{128\pi\lambda} \right)^{\frac{2}{n+2}}. \quad (27)$$

For $n = 4$, the number of e-folds for the chaotic eternal inflation must be as large as $N_* \simeq 4.4 \times 10^4$. According to eq. (20), the tunnelling probability is extremely suppressed with $\mathcal{F}_{N_*} \simeq 4 \times 10^4 \ll \mathcal{F}_{60}$. The tunnelling probability for the chaotic inflation starting with ϕ_* or larger is negligible. Therefore the improved Hartle-Hawking wave function avoids the chaotic eternal inflation.

In fact, there is a debate about the correct sign in the exponent in the tunnelling probability from nothing to a closed de Sitter universe. In [17], the authors proposed that the tunnelling probability is $\exp\left(-\frac{3}{8\Lambda}\right)$. It prefers a large cosmological constant and the chaotic eternal inflation naturally emerges. Recently the authors in [18] pointed out that the decoherence effect enhances the tunnelling probability if we use the results in [17]. It seems quite unreasonable. It is possible that the improved Hartle-Hawking wave function can lead to some observational results.

We also notice that the improved Hartle-Hawking wave function (22) prefers a universe with the total number of e-folds as small as possible. This is a very subtle point. It means a closed universe with large enough spatial curvature to be detected can naturally emerge even for chaotic inflation. It is quite different from the traditional point of view [19]. In fact, the WMAP teams find that the best fit model to the first year data of WMAP combined with other data sets is slightly closed with $\Omega_k = -0.02 \pm 0.02$ in [1], where $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$. Other results are all consistent with a slightly closed universe (for example, see [20]). Additionally, the WMAP results [1] confirm

that the amplitude of the quadrupole in the temperature power spectrum is low compared with the predictions of Λ CDM models seen by COBE [21] as well. In [22], Efstathiou proposed that the low CMB quadrupole amplitude may be related to a truncation of the primordial fluctuation spectrum on the curvature scale in a closed universe. But it is very difficult to obtain a realistic model of a closed inflationary universe before. Here we find that the improved Hartle-Hawking wave function can give us a possible mechanism to provide a detectable spatial curvature and help us to explain the deficit of quadrupole in the temperature power spectrum for chaotic inflation if the gravitational UV cutoff M or string scale M_s in string theory is roughly $0.02M_{pl}$.²

The scalar power spectral index n_s and the tensor-scalar ratio r is still not clear. Current measurements suggest that $n_s = 0.980 \pm 0.020$ with 68% confidence and $r \leq 0.36$ with 95% confidence by combining WMAP and the SDSS galaxy survey [23]. A fit using WMAP and BOOMERANG CMB data and the SDSS and 2dFGRS galaxy surveys [24] gives $n_s = 0.950 \pm 0.020$ at 68% confidence level. The tensor-scalar ratio for the chaotic inflation is given by $r = 4n/N \sim \mathcal{O}(10^{-1})$ and the spectral index is $n_s = 1 - \frac{n+2}{2N}$ ($n_s \simeq 0.967$ for $n = 2$ and $n_s \simeq 0.95$ for $n = 4$). Both the spectral index and the tensor-scalar ratio for the chaotic inflation are close to the observational bounds. We expect the cosmological observations can provide stronger evidence to support or rule out this model in the near future.

Similarly we also investigate the inflation models with two fine tuning parameters, for example, brane inflation type model with potential

$$V(\phi) = V_0 \left(1 - \frac{\mu^n}{\phi^n} \right), \quad (28)$$

where $n \geq 1$. There are two parameters, V_0 and μ . The evolution of inflation

²There are also many other works on the deficit of the quadrupole, see [25].

is dominated by V_0 and the tunnelling probability from nothing is

$$P \simeq \exp \mathcal{F} = \exp \left(\frac{3}{8V_0} - \frac{27\nu}{32V_0^2} \right). \quad (29)$$

The tunnelling probability goes to its maximum value when

$$V_0 = V_{0m} = \frac{9\nu}{2}, \quad (30)$$

with

$$P \simeq \exp \mathcal{F} = \exp \left(\frac{1}{24\nu} \right) = \exp \left(\frac{3}{16V_0} \right). \quad (31)$$

The tunnelling probability at V_0 close to V_{0m} is negligible compared to that at V_{0m} . If the amplitude of the power spectrum is taken as an input, we obtain a relationship between V_0 and μ . Thus there is only one free parameter. The equations of motion can be written down as

$$H^2 \simeq \frac{8\pi V_0}{3}, \quad (32)$$

$$3H\dot{\phi} \simeq -\frac{nV_0\mu^n}{\phi^{n+1}}. \quad (33)$$

The number of e-folds before the end of inflation is

$$N = \int H dt = \frac{8\pi}{n(n+2)} \frac{\phi_N^{n+2}}{\mu^n}. \quad (34)$$

Or equivalently, the value of ϕ , namely ϕ_N at the number of e-folding N before the end of the inflation is given by

$$\phi_N = \left(\frac{n(n+2)}{8\pi} \mu^n N \right)^{\frac{1}{n+2}}. \quad (35)$$

The amplitude of the power spectrum can be expressed as

$$\delta_H \simeq \left(\frac{2^9 \cdot \pi}{3 \cdot 5^2 \cdot n^2} \right)^{\frac{1}{2}} \left(\frac{n(n+2)}{8\pi} \right)^{\frac{n+1}{n+2}} \frac{V_0^{\frac{1}{2}}}{\mu^{\frac{n}{n+2}}} N^{\frac{n+1}{n+2}}. \quad (36)$$

For $n = 2$, using COBE normalization, we obtain

$$V_0 = 9 \times 10^{-13} \mu, \quad (37)$$

and $\mathcal{F} \simeq 2.1 \times 10^{11}/\mu$. If ν is given, the value of parameter μ is determined by using eq. (30) and (37). On the other hand, smaller the parameter μ , or equivalently lower the inflation energy scale, larger the tunnelling probability. The amplitude of the tensor (gravitational wave) fluctuations only depends on the inflation energy scale. According to the improved Hartle-Hawking wave function, an inflation model with small amplitude of the primordial tensor fluctuations is favored. We can expect that the tensor-scalar ratio is too small to be detected for this inflation model. We also notice that there is no stringent constraint on the number of e-folds and then the spatial curvature should be neglected. The spectral index for this model is $n_s = 1 - \frac{n+1}{n+2} \frac{2}{N}$ ($n_s \simeq 0.975$ for $n = 2$ and $n_s \simeq 0.972$ for $n = 4$), a red spectrum as well. The similar results are also obtained for the inflation model with potential $V(\phi) = V_0(1 - \phi^n/\mu^n)$, where $n > 2$.

In summary, the improved Hartle-Hawking wave function offers many interesting observational consequences. It naturally provides a possible mechanism, otherwise hard for us to understand before, to obtain a slightly closed universe with a detectable spatial curvature for chaotic inflation, which is consistent with the current results of the cosmological measurements. However the observational consequences for the brane inflation type model are very boring, no detectable spatial curvature and tensor fluctuations, and more fine tuning parameters are included. We expect that the cosmological observations will give us more conclusive results in the near future.

An old but very important problem in Hot Big Bang model is where the initial expansion rate of the universe came from. Quantum cosmology give us a scenario for the universe with initial expansion rate being zero. A positive cosmological constant provide the force to push the expansion of the universe. When the positive cosmological constant dominated the evolution of the universe, the expansion of the universe is accelerating, namely inflating, and the expansion rate is quite large. At the end of the inflation, Hot Big Bang happened. This is roughly the history of the universe. However, there

is still one more question: where does the cosmological constant come from, which is still hard for us to answer. Maybe tension of the branes in string theory is a nice candidate for the positive cosmological constant.

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References

- [1] C. L. Bennett et al., *Astrophys. J. Suppl.* 148 (2003) 1, astro-ph/0302207; D. N. Spergel et al., *Astrophys. J. Suppl.* 148 (2003) 175, astro-ph/0302209; D. N. Spergel et al., astro-ph/0603449.
- [2] A. G. Riess et al., *Astron. J.* 116 (1998) 1009, astro-ph/9805201; S. Perlmutter et al., *Astrophys. J.* 517 (1999) 565, astro-ph/9812133; A. G. Riess et al., *Astrophys. J.* 607 (2004) 665, astro-ph/0403292; Y. Wang and M. Tegmark, *Phys. Rev. Lett.* 92 (2004) 241302, astro-ph/0403292.
- [3] J. B. Hartle and S. W. Hawking, *Phys. Rev. D* 28 (1983) 2960.
- [4] A. Vilenkin, *Phys. Lett. B* 117 (1982) 25; A. Vilenkin, *Phys. Rev. D* 27 (1983) 2848.
- [5] A. A. Starobinsky, *Grav.Cosmol.* 6 (2000) 157, astro-ph/9912054.
- [6] K. Dasgupta, G. Rajesh and S. Sethi, *JHEP* 9908 (1999) 023, hep-th/9908088; J. L. Feng, J. March-Russell, S. Sethi and F. Wilczek, *Nucl. Phys. B* 602 (2001) 307, hep-th/0005276; S. B. Giddings, S. Kachru and J. Polchinski, *Phys. Rev. D* 66 (2002) 106006, hep-th/0105097; S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, *Phys. Rev. D* 68 (2003) 0406005, hep-th/0301240; F. Denef, M. R. Douglas and B. Florea, *JHEP* 0406 (2004) 034, hep-th/0404257; F. Denef and M. R. Douglas, *JHEP* 0405 (2004) 072, hep-th/0404116.
- [7] A. Vilenkin, *Phys. Rev. Lett.* 74 (1995) 846, gr-qc/9406010.
- [8] H. Firouzjahi, S. Sarangi and S. H. Henry Tye, *JHEP* 0409 (2004) 060, hep-th/0406107.
- [9] S. Sarangi and S. H. Henry Tye, hep-th/0505104.
- [10] E. Verlinde, hep-th/0008140.
- [11] A. Linde, *JCAP* 0410 (2004) 004, hep-th/0408164.
- [12] B. McInnes, hep-th/0509035.

- [13] E. W. Kolb and M. S. Turner, *The early universe*, Addison-Wesley Publishing Company.
- [14] A. Linde, Phys. Lett. B 129 (1983) 177.
- [15] A. Vilenkin, Phys. Rev. D 27 (1983) 2848; A. D. Linde, Phys. Lett. B 129(1986)395.
- [16] A. D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood Academic, Chur, 1990, hep-th/0503203.
- [17] A. Linde, Nuovo Cim. Lett. 39(1984)401; A. Vilenkin, Phys.Rev.D 30(1984)509.
- [18] S. Sarangi and S. Tye, hep-th/0603237.
- [19] A. Linde, JCAP 0305 (2003) 002, astro-ph/0303245.
- [20] M. Tegmark et al., Phys. Rev. D69 (2004) 103501, astro-ph/0310723.
- [21] G. Hinshaw, ApJ 464 (1996) L17.
- [22] G. Efstathiou, Mon.Not.Roy.Astron.Soc. 343 (2003) L95, astro-ph/0303127.
- [23] U. Seljak, et al., Phys. Rev. D 71 (2005) 103515, astro-ph/0407372.
- [24] C. J. MacTavish et al., astro-ph/0507503.
- [25] K. Enqvist and S. Sloth, Phys. Rev. Lett. 93 (2004) 221302; K. Enqvist, S. Hannestad and S. Sloth, JCAP 0502 (2005) 004; Qing-Guo Huang and M. Li, JHEP 0306 (2003) 014, hep-th/0304203; Qing-Guo Huang and M. Li, JCAP 0311 (2003) 001, astro-ph/0308458; Qing-Guo Huang and M. Li, Nucl. Phys. B 713(2005)219, astro-ph/0311378; S. Tsujikawa, R. Maartens and R. Brandenberger, Phys. Lett. B 574 (2003) 141; B. Feng and X. M. Zhang, Phys. Lett. B 570 (2003) 145; Y. S. Piao, B. Feng and X. M. Zhang, Phys. Rev. D. 69 (2004) 103520; C. R. Contaldi, M. Peloso, L. Kofman and A. Linde, JCAP 0307 (2003) 002; G. Calcagni, S. Tsujikawa, Phys.Rev. D70 (2004) 103514, astro-ph/0407543.